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Mapping whole-event drive losses: the impact of race profile and rider input on transmission efficiency in cycling

George C Barnaby*, **Stuart Burgess and Jason Yon**

Department of Mechanical Engineering, Queen's Building, University of Bristol. BS8 1TR. UK

* Correspondence: (GB) george.barnaby@bristol.ac.uk

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Abstract: Several studies have considered the factors influencing transmission efficiency in a bicycle. These conclude that the effective radius of sprockets engaged with the chain and the torque and cadence of the cyclist influence the frictional losses associated with transmission between rider and rear wheel. These parameters may vary significantly during a bicycle race since a rider modifies their gear, power, and cadence to maximise physiological efficiency for optimum bicycle velocity. Furthermore, gearing selection and power input varies between riders, riding group and course profile. However, power models used to estimate race outcomes tend to simplify efficiency to a single, arbitrary factor, describing losses which scale linearly with input power regardless of expected regime. This study extends existing analytical descriptions of transmission losses to the context of a road bicycle with front and rear derailleurs. The calculated efficiency is considered within a cycling model to judge different regimes under which the chain will typically operate and maps overall performance during an event. Efficiency may vary significantly under certain loading regimes shown. In the context of highly trained cyclists these differences result in small changes which are symmetrical about a mean value. This study shows there is limited error in assuming constant efficiency for certain race types, though the efficiency value itself is dependent on several factors affecting the average loading regime. Elevation profile of the racecourse and average power input from the rider are key parameters affecting average efficiency. More massive riders racing at high average power input will experience higher efficiency, while efficiency is higher across all riders racing courses with increased elevation gain.

Keywords: transmission; efficiency; model; losses; bicycle; derailleur.

1. Introduction

In cycling, the use of analytical models to describe the balance of input power at the crank and output power at the tyre-road interface allows the engineer to identify areas for improvement in rider technique or equipment design. One such example is described in equation (1), based on an analytical model from Martin et al, 1998.

$$P_{in} = V(F_a + F_r + F_g)/\eta, \qquad \text{Eq. (1)}$$

where P_{in} is the input power of the rider; F_a, F_r and F_g are the resistive forces associated with aerodynamic drag, rolling resistance between tyre and road, and gravitational resistance; V is the bicycle velocity; and η is the transmission efficiency.

In deriving this model, and commonly in literature, transmission efficiency is assumed to be a constant value such that losses scale linearly with power input, and often an arbitrary estimate. Later studies, however, demonstrate that the same chain in a derailleur transmission system has measured efficiency in the range 80.9 – 98.6% (Spicer et al., 2001). The changing factors causing this range in efficiency are input power, rotational speed, and gear configuration, which may also vary greatly during a bicycle race: gear shifts and non-consistent physiological output from the rider are common consequences of varying road-race profiles.

There is a gap in published literature for a holistic consideration of the efficiency to study these dependencies in the context of different racecourses and riders, which may be useful in determining the error in assuming constant efficiency and providing recommendations for what efficiency estimate to use for riders and engineers based on course and rider profile.

This study seeks to investigate the variability of transmission efficiency in expected regimes and defines the key factors influencing the transmission efficiency in usable terms, such that riders and engineers might be better informed in their use of an estimated efficiency in future modelling.

2. Frictional loss model

The authors are unaware of a comprehensive model of frictional losses in a bicycle derailleur drive in literature, and so have derived an analytical approach. This is an extension of the work of Lodge & Burgess, 2001, as is used in Barnaby et al., 2020.

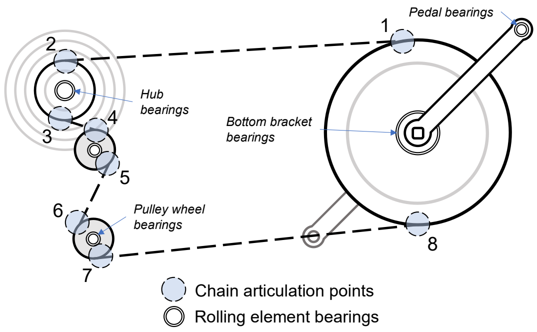

Figure 1. Sources of friction in a bicycle transmission, including rolling element bearings and points of chain articulation (numbered).

To determine the relative contribution of different sources of friction, analysis from Lodge & Burgess is used in conjunction with the geometry and spring rate of the rear derailleur to predict low-tension span tension. An industrial model of bearing losses is used to estimate friction in bearings (SKF, n.d.).

The relative losses of each of the sources of friction, shown in Figure 1, is summarised in Table 1. There is significant contribution to losses of the high-tension span articulations (71%), a smaller contribution from low-tension span articulations (26%), and a near-negligible contribution from rolling element bearings (3%). Friction in rolling element bearings is henceforth neglected in this analysis.

Table 1. Power losses are approximated for different sources of friction in the drive (300W / 90rpm)

	Power loss [W]	% of total
High-tension span [1]	5.5	71
Low-tension span [2]	2.0	26
Rolling element bearings	0.2	3
Total	7.7	100

[1] Chain articulations 1-2; [2] chain articulations 3-8.

Transmission efficiency is defined as in equation (2):

$$\eta = \left(P_{in} - N_s \omega_s \sum_{i=1}^{8} W_i\right)/P_{in}, \quad \text{Eq. (2)}$$

where W is the work done against friction in each of 8 articulating links (entry and exit to each sprocket), ω_s is the rotational frequency of the chainring (s^{-1}) and N_s is number of teeth in the chainring. Work done against friction is a function of chain geometry, articulation angle and chain tension, all of which may be calculated based on specific equipment and rider input. A further dependency is on coefficient of sliding friction within the chain links, which can be accurately determined experimentally using techniques such as those proposed by Wragge-Morley et al., 2017. The calculation for work done against friction is included in Appendix I.

2.1 Transmission efficiency variation

The variation of transmission efficiency is examined over a range of cycling torque inputs and riding gears, shown in Figure 2. The low, hill climbing gears offer higher efficiency due to the reduced articulation angle. Positive correlation between input torque at the crank and efficiency is due to the relative reduction of significance of the low-tension span losses, which are independent of torque input. At low torque the torque-independent losses in the low-tension span, tensioned by the derailleur arm, are relatively more significant and so transmission efficiency changes rapidly as a function of torque.

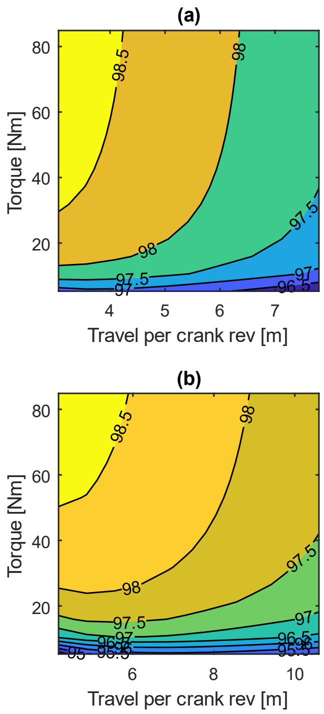

Figure 2. Power efficiency [%] contour map for varying rider torque and gear for 11-28 tooth cassette sprockets engaged with (**a**) 39-tooth chainring; and (**b**) 53-tooth chainring.

3. Variable efficiency within power model

To model how transmission efficiency varies in a race, simulation of typical power input and race profile is necessary since efficiency depends on power and gear selection, themselves having multivariant dependencies. The power required to overcome resistance at steady speed cycling is given in equations (3) – (6), based on work by Martin et al., 1998.

$$P_{in} = \left(P_a + P_r + P_g\right)/\eta, \qquad \text{Eq. (3)}$$

where power to overcome aerodynamic drag, P_a, is described in equation (4), power to overcome rolling resistance of the tyres, P_r, is described in equation (5) and power to overcome gradient, P_g is described in equation (6).

$$P_a = 0.5\rho C_d A_f V^3, \qquad \text{Eq. (4)}$$

where ρ is air density, C_d is coefficient of aerodynamic drag, A_f is the frontal area of bicycle and rider, and V is bicycle velocity. Note that wind velocity is assumed to be zero in this analysis.

$$P_r = mgC_{rr}V, \qquad \text{Eq. (5)}$$

where m is total mass of rider and bicycle, g is the gravitational acceleration constant and C_{rr} is the coefficient of rolling friction between tyre and road surface. Upright, straight-line cycling is considered for this analysis.

$$P_g = mg\sin(\theta)\,V, \qquad \text{Eq. (6)}$$

where θ is the angle of gradient. Typical values for variables shown in equations (4) – (6) are from Wilson, Papadopoulos, and Whitt, 2004.

Steady-state velocity is calculated at many discrete points along a simulated racecourse for a typical bicycle drivetrain. Gearing is selected to maintain cadence within a typical range, with chosen gearing influencing the calculation for efficiency according to the described frictional loss model. Further, a variable power input is applied such that power increases with positive gradient and decreases with negative gradient, shown to be an effective pacing strategy (Wells & Marwood, 2016).

3.1 Efficiency variation by rider type

Transmission efficiency is simulated for an example elite race: part of the UCI 2021 World Road Championships road-race from Antwerp to Leuven (UCI, 2021). Efficiency is illustrated for four different riders in Figure 3, where rider input parameters are summarised in Table 2. The spread of efficiency estimates during simulated race for each case may be seen in Figure 4.

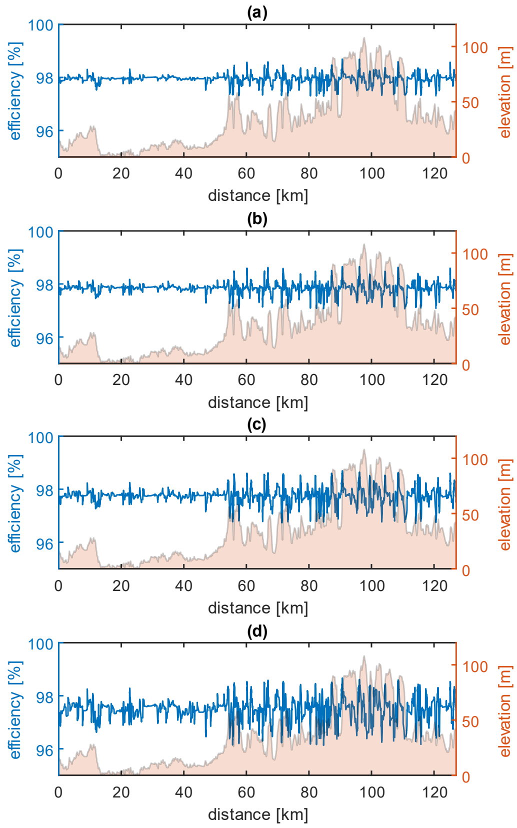

Figure 3. Transmission efficiency overlayed on Leuven 2021 road-race course profile for a (**a**) elite male cyclist, (**b**) elite female cyclist, (**c**) untrained male cyclist, and (**d**) untrained female cyclist.

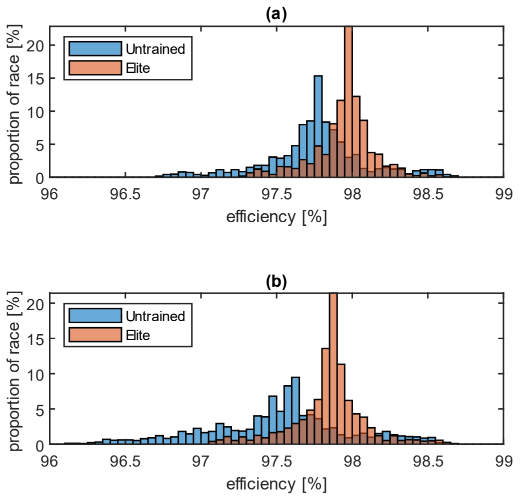

Figure 4. Histogram of simulated transmission efficiency during example race for elite and untrained riders.

Efficiency can be seen to fluctuate with the gradient of the course due to the changing gear and power. Hill climbing gears and a marginal increase in power both result in increased efficiency as has been shown previously. The opposite is true for negative gradients, where smaller sprocket is engaged and power is slightly reduced, decreasing efficiency.

Table 2. Input parameters for four modelled cases, with estimates for elite and untrained male and female riders.

	Elite		Untrained	
	Male	Female	Male	Female
Mass [kg]	70	60	80	65
C_dA [m^2]	0.3	0.25	0.4	0.3
Average power [W]	350	250	150	100
Cadence [rpm]	90±10	90±10	70±15	70±15
Average efficiency (S.D.) [%]	98.0 (0.20)	97.9 (0.24)	97.8 (0.33)	97.6 (0.46)

Comparing the proficient and untrained cases, a larger variance can be seen in the untrained cyclist as well as a slightly lower average efficiency. This is illustrated more clearly in Figure 4. The lower average efficiency is largely due to the reduced power input, and hence lower average torque. The variance is reduced in the trained cyclist due to the responsive gear changes working to maintain a high cadence.

3.2 *Efficiency variation in elite riders*

In elite level racing, efficiency variance during a race is low and there is little error in determining average velocity, or time to completion, by using a single value efficiency across an entire race. This is determined in simulated races by finding the ratio of total energy input and total energy output, found by integrating the power output and input with respect to time as in equation (7).

$$\eta = \frac{\int P_{in} - P_{lost}\,dt}{\int P_{in}\,dt}, \qquad \text{Eq. (7)}$$

However, there is still dependency of this average efficiency on power input and gearing, which itself is dictated by the elevation profile of a racecourse.

An effective average efficiency is determined by interpolating between results from simulations with varying parameters. Transmission efficiency during 20 different grand tour events is modelled with riders of varying input power (50-550W) and mass (50-80kg). Dependency of average efficiency on the climbing during the race (measured by average metres elevation gain per kilometre travel), and the average power achieved by the rider during the race is illustrated in Figure 5. Mass is less impactful and can be accounted for by applying an additional 0.1% efficiency per 20kg above 65kg. These results are valid for riders with a power-to-weight ratio of between 2 and 6 W/kg.

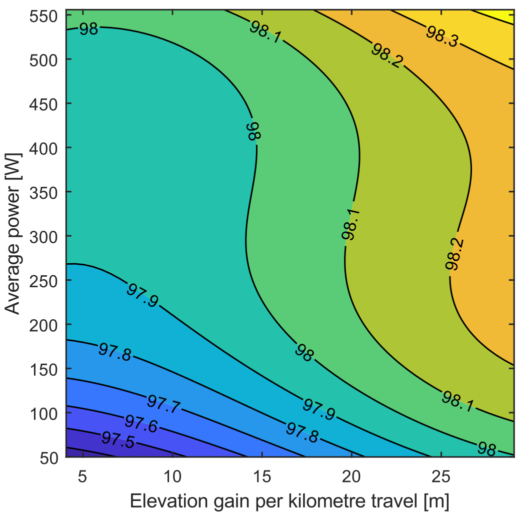

Figure 5. Contour map of example transmission power efficiency [%] as function of average power during a race and its elevation profile.

4. Discussion

The range in efficiency found in previous research is not realised in loading regimes typical in elite racing. This is largely because of the narrow cadence range and high torque in elite racing which results in a low variance, symmetrically distributed spread of transmission efficiencies about a mean value. Provided conditions and input parameters are maintained during a race, there is little error in using a single factor for efficiency.

However, average power and elevation profile are two factors which can vary greatly in elite cycling between different event styles and rider physiologies, leading to consistent changes to efficiency across a race. A mountainous tour stage will see higher efficiency than one which is flat by up to 0.5%-pts, which may be even more extreme if considering specific hill climbing events. Rider power input also will influence average efficiency, meaning that more powerful male riders racing TT courses at maximal effort may experience an average transmission efficiency up to 0.8%-pts higher than a less powerful rider during an endurance event. Female elite riders will inherently experience a reduced transmission efficiency than male elite riders due to applying less power at the crank.

5. Practical Applications

Transmission efficiency may usually be experimentally examined in limited and specific loading regimes. This gives limited insight given the dependencies of efficiency which vary during a bicycle race. This study demonstrates that further applying the results of such tests to contextualise efficiency within the expected loading regimes based on rider and course type may offer additional accuracy in determining an effective average efficiency. This may be applied to future analytical modelling for evaluating equipment upgrades or determining pacing strategies.

Future research to further examine influences on transmission efficiency is needed to confirm the theory presented here, including extensive practical testing which may offer experimental validation.

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References

1. Barnaby, G. C., Yon, J., & Burgess, S. (2020). Sprocket Size Optimisation for Derailleur Racing Bicycles. *Journal of Science and Cycling*, *9*(2), 36.
2. Lodge, C. J., & Burgess, S. C. (2001). A model of the tension and transmission efficiency of a bush roller chain. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, *216*(4), 385-394.
3. Martin, J. C., Milliken, D. L., Cobb, J. E., McFadden, K. L., & Coggan, A. R. (1998). Validation of a mathematical model for road cycling power. *Journal of applied biomechanics*, *14*(3), 276-291.
4. SKF model for calculating the frictional moment (n.d.). Retrieved from https://www.skf.com/binaries/pub12/Images/0901d1968065e9e7-The-SKF-model-for-calculating-the-frictional-moment_tcm_12-299767.pdf
5. Spicer, J. B., Richardson, C. J., Ehrlich, M. J., Bernstein, J. R., Fukuda, M., & Terada, M. (2001). Effects of frictional loss on bicycle chain drive efficiency. *J. Mech. Des.*, *123*(4), 598-605.
6. UCI 2021 Road World Championships Flanders Belgium (2021). Retrieved from https://www.flanders2021.com/en/races
7. Wells, M. S., & Marwood, S. (2016). Effects of power variation on cycle performance during simulated hilly time-trials. *European journal of sport science*, *16*(8), 912-918.
8. Wilson, D. G., Papadopoulos, J., & Whitt, F. R. (2004). *Bicycling science (3rd ed.)*. MIT press.
9. Wragge-Morley, R., Yon, J., Lock, R., Alexander, B., & Burgess, S. (2018). A novel pendulum test for measuring roller chain efficiency. *Measurement Science and Technology*, *29*(7), 075008.

Appendix I

Work done against friction in articulating chain links as derived by Lodge & Burgess, 2001, is summarised here. Eight articulations are considered, at entry to and exit from each engaged sprocket. The work done in articulating these links simultaneously represents the energy lost for the drive advancing by one link, which may be multiplied by the chain speed in link pitch per second to determine the power lost here.

Work done in articulating chain links

The articulation of inner and outer links results in relative sliding of different surfaces between the pin and bushing. Since they must alternate, an average is taken of the two to define work done for one articulation:

$$W = \left(W_{pin} + W_{bush}\right)/2$$

where work done during pin articulation is described as below:

$$W_{pin} = \frac{F_i}{\sqrt{1+\mu^2}}\mu r_{bi}\alpha_i$$

and work done during bush articulation is:

$$W_{bush} = \frac{\mu F_c r_{bi}\left[\cos\theta_{RA} - \cos(\theta_{RA}+\alpha_m)\right]}{\sqrt{1+\mu^2}\sin(\theta_{RA}+\alpha_m)} + \frac{\mu F_c r_{bo}(1-\cos\alpha_m)}{\sin(\theta_{RA}+\alpha_m)}$$

Lodge & Burgess, 2001 should be consulted for definitions of these terms. These are determined by the geometry of the chain components and sprocket, except for coefficient of friction, μ, which is determined using accurate measurements as described in Wragge-Morley et al., 2017.

Chain tension force

The contact force between each of 8 articulating links is calculated. The high-tension span contact force is from crank torque acting at chainring radius and acts at articulations onto the chainring and off the engaged cassette sprocket. Low-tension span contact force is assumed to be equal for all 6 remaining articulations and resolved from the estimated spring rate of the rear derailleur tension arm and its geometry.