RESEARCH ARTICLE

# The Elimination Race in Track Cycling: Patterns and Predictors of Performance

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## Abstract

The track cycling Omnium is a multi-event competition that has recently been expanded to include the Elimination Race (ER), which presents a unique set of physical and tactical demands. The purpose of this research was to characterise the performance attributes of successful and unsuccessful cyclists in the ER, that are also predictive of performance. Video recordings of four international level ERs were analysed. The performance attributes measured related to the cyclists' velocity and two dimensional position in the peloton. The average velocity of the peloton up to lap 30 (of 50) was relatively high and consistent (52.2±1.5 km/h). After lap 30, there was a significant (p<0.001) change in velocity (49.9±2.4 km/h), characterised by more fluctuations in lap-to-lap velocity. Successful ER cyclists adopted a tactic of remaining in the middle of the peloton, in the lower lanes of the velodrome, thus avoiding the risk of elimination at the rear and the extra effort required to remain on the front of the peloton. Unsuccessful cyclists tended to reside in the rear and upper (higher) portions of the peloton, risking elimination more often and having to ride faster than those in the lower lanes of the velodrome. The physiological demands of the Elimination Race that are determined by velocity, vary throughout the Elimination Race and the pattern of movement within the peloton is different for successful and unsuccessful cyclists. The findings of the present study may confirm some aspects of race tactics that are currently thought to be optimal, but they also reveal novel information that is useful to coaches and cyclists who compete in the Elimination Race.

**Keywords:** performance analysis, multi-event, statistical analysis, machine learning

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## Introduction

The Omnium is a multi-event track cycling competition that was first introduced by the International Cycling Union (UCI) in 2007. Cyclists accumulate Omnium points in accordance with each cyclist's finishing rank in a series of events (highest ranks are given lowest points). Cyclists win the Omnium by completing all events with the lowest total Omnium points. The demands of each event vary across a range of fitness profiles (sprint to endurance) as well as tactical and technical abilities (individual time trials vs. mass start races). Thus to win the Omnium, the cyclist must possess a broad range of cycling abilities.

In 2009 the Omnium changed in a variety of ways, the distance of most events increased and the duration of the competition was increased from one to two days. Perhaps, the most important change was the increase in the number of events from five to six, with the addition of the Elimination Race (ER). The ER is a new style of race that has not been previously conducted in UCI championships. After a group of up to 24 cyclists complete a rolling start, on every second lap for up to 50 laps, the last cyclist across the finishing line is eliminated from the race. The last remaining cyclist is deemed the winner of the ER and Omnium points are awarded in the order that each cyclist was eliminated from the race.

The ER race is challenging for cyclists because of its novelty. Very few cyclists or cycling coaches have any long term experience of the ER at elite levels of competition and it is unlike the other Omnium events. The most similar event may be the Points Race, in which cyclists compete to win sprints every ten laps in order to accumulate points. However, the Points Race is much longer than the ER (120 and 80 laps for men and women respectively) and the requirement for sprint efforts is far less frequent (every 10 laps vs. every 2 laps). An additional significant challenge of the ER is its tactical demands on each cyclist. Because the primary goal of the ER is to avoid being at the back of the peloton, rather than trying to be at the front, which is typical of most races (e.g. Points Race), the dynamics of the peloton appear to be different to other mass start races. In addition, the race is one of "sudden death" where finishing a sprint last, early in the race, eliminates that cyclist and establishes his or her rank for this event, unlike the Points Race where early mistakes can be compensated for later in the race.

As further evidence of the novel and unusual demands of the ER, previously conducted analysis of the Omnium, has revealed the typical patterns of performance in each of the events that are required to win a medal in the Omnium overall (Ofoghi et al. 2012). This analysis revealed correlations between performance in several of the Omnium events like the Individual Pursuit (r=0.77) and Time Trial (r=0.79), with overall rank in the Omnium. The authors were able to show with some certainty, what kinds of performances were required in each of the Omnium events to create a high likelihood of winning a medal in the Omnium. However, in the same study, rank in the ER had only a low correlation (r=0.59) with overall rank in the Omnium, which suggests that the ER has a unique set of demands that make it very challenging for cyclists. Indeed, its novelty may mean that cyclists and their coaches have not yet developed a shared understanding of the ideal race tactics. If this is the case, then the dynamics of the race may evolve for the next few years, which makes it inherently interesting to examine.

Success in most sporting competitions is determined by a complex set of factors relating to the performance of the athlete, their team, their opposition and the interaction of their tactics and the rules of the event. Many athletes and coaches at the elite level understand a lot of the basic determinants of success in their sport, but few could claim they possess a model that accurately and comprehensively relates athletic performance to the likelihood of success. In an era when many aspects of athletic performance can and are being measured, Data Mining techniques can be used to interrogate large databases of sport performance information in the pursuit of two main goals; the identification of characteristics and/or patterns of winning performances, and the provision of information to support tactical decision making during an event, via real time analysis of sport performance.

The purpose of the present work was to characterise the performance of the best and worst cyclists in Elimination Race. Specifically, this work sought to describe the changes in the velocity of the peloton through the ER, identify the movement patterns of the best and worst cyclists within the peloton and use data mining techniques to identify performance characteristics that are associated with successful outcomes.

## Materials and methods
### Participants
The men's Elimination Races at the Melbourne (Dec 2010), Beijing (Jan 2011) and Manchester (Feb 2011) UCI (International Cycling Union) track cycling World Cups and the 2011 UCI World Championships, held in Apeldoorn, Netherlands (March 2011) were analysed in this study. The four races analysed involved 91 cyclists, 66 of whom were unique as some cyclists raced in multiple World Cups or Championships.

### Procedures
*Video Collection and Analysis*
Ethical approval for this project was provided by the Human Research Ethics Committee at Victoria University. Video recordings of the ERs were analysed using Kinovea video analysis software (v 0.8.15, http://www.kinovea.org). All races were filmed from the stands above the back straight and second bend, in line with the start/finish line.

Using the final placing of each cyclist, we performed a retrospective performance analysis. For each two-lap elimination cycle, we analysed the performance of the cyclists who finished the ER in first, second and third places, as well as the eliminated cyclist and the cyclist who was to be eliminated in the next two-lap elimination cycle. Thus, we could describe the performance of each eliminated cyclist for the four laps prior to their elimination. Data were collected from the video on every lap, when the cyclist placed last in the peloton, crossed the finish line. The variables collected for analysis include lap time, horizontal and vertical position in the peloton, rank in the peloton and distance behind the front of the peloton. Table 1 provides a list of variables and definitions.

**Table 1.** The performance attributes and lap velocity metrics identified in the analysis of the Elimination Races. The lap times and lap velocities were taken using the stopwatch feature in Kinovea video analysis software. Lap times (ss.00) were taken for each cyclist in question and were timed as the back wheel of that cyclist crossed the start/finish line each lap. See Figure 1 for a description of position measurements.

| Variable | Definition |
| --- | --- |
| **Elimination lap #** | Lap number that the rider was eliminated |
| **# of cyclists** | The number of cyclists remaining in the peloton |
| **Lap# time (s)** | The number of seconds taken to complete the lap |
| **Lap# velocity (km/h)** | Calculated from lap time and lap distance (250 m) |
| ***Position*** | |
| **Lap# x** | Horizontal distance in bike lengths from the cyclist in last position |
| **Lap# y** | Vertical height on the track, with cyclists below the upper line of the sprinter's lane being given a 'y' coordinate of 1 and cyclists at the highest point on the track being given a 10. Y values were assigned in terms of approximate bike widths through visual inspection of the video and markings on the track. |
| **Lap# pos** | Position in the peloton ( ie. 1st, 3rd, 14th, etc) |
| **Lap# pos_n** | Division of the cyclist's position by the number of cyclists in the pack (i.e. 0.99 is last place in a peloton of any size) |
| **Lap# distance to first** | Horizontal distance in bike lengths from the cyclist in first position at that point in the race |

If a cyclist was out of frame of the video, their data point/s remained missing from the data set. If it was possible to accurately estimate the cyclist's position (for example, if he was partially in frame or had just moved out), this was done at the researcher's discretion. Two of the authors completed all of the video analysis, by initially working together to develop the coding methodology and procedure. They crossed checked their coding to ensure coherence and then proceeded to analyse all of the video data using a consistent method.

### Identification of stages within the ER

Given the progressive decrease in the size of the peloton throughout the ER, we have observed changes in the demands on (i.e. changes in lap velocity) and the dynamics within the peloton during the race. Our subjective assessment was that there may be two or three stages within the ER, each with its own unique combination of demands. Therefore we explored this possibility by evaluating changes in the only variable we measured that can be reasonably compared for the entire duration of the race; lap-to-lap peloton velocity.

To determine the possible existence and approximate position of stages in the ER, we used an unsupervised machine learning method. Machine learning is that subsection of learning in which the artificial intelligence system attempts to learn automatically. In unsupervised learning, the system receives only the input, and no information on the expected output. The system learns to produce the pattern to which it has been exposed. The clustering method we used was the well-known $k$-means algorithm (MacQueen 1967). The aim of clustering techniques is to group data into clusters of similar items. We used the WEKA machine learning package (MacQueen 1967) to run the $k$-means algorithm and the *elbow* method to estimate the best number of clusters in the ER dataset (Mardia et al. 1979). The optimal number of clusters was identified, when adding another cluster did not achieve better modelling of the data. For this, we calculated the *within cluster sum of squared errors* (WCSSE) of cluster analysis with the number of clusters. For higher reliability of the results, we utilised two kernel functions for calculating WCSSE values using $k$-means, namely the *Euclidean* distance and *Manhattan* distance functions. The ER data used for this analysis were average, standard deviation, minimum and maximum velocity of riders in each of the four lap analysis cycles.

### Performance modelling and prediction

We carried out performance modelling and prediction in three steps: i) developing a classification system by considering all related ER attributes, ii) selecting the attributes most related to performance that can be used for elimination race performance modelling and prediction, and ii) modelling a classification system for predicting cyclists' performances based on the optimal set of attributes.

### Analysing all performance attributes

To find a systematic way of predicting the cyclists' performances, in terms of whether they will win a medal or not in the Omnium (i.e., finish the ER in a certain rank), we utilised a supervised machine learning technique. Classification is a method that predicts group memberships for data instances (individual cases) in a dataset. A previous study on the ER in the context of the track cycling Omnium has shown that the Omnium (male) medal winners finish the ER in 6th place on average (Ofoghi et al. 2012). We took this ER rank as the boundary for defining successful and unsuccessful ER riders who compete in an Omnium.

We used the ER dataset with all of the performance attributes described earlier (Table 1.) excluding attributes that would not be of obvious use; number of riders in the peloton and the lap number on which a rider was eliminated. We pre-processed the dataset by converting the ER ranks into only two categories: i) ER rank=1 for all ER rank≤6 and ER rank=2 for all ER rank>6. These two ER rank categories represent successful cyclists and unsuccessful cyclists for this performance prediction analysis. We utilised the Naïve Bayes classification method (George and Langley 1995) in WEKA to model an automated classifier that can assign a class label (i.e., successful or unsuccessful) to a performance data record comprising the above-mentioned performance attributes.

### Identification of the most predictive performance attributes

Given the large number of related performance attributes used for modelling and prediction described in the previous section, we used a number of machine learning-based feature selection methods to find the most relevant attributes. The techniques we used were correlation-based feature subset selection (Hall 1998), gain ratio evaluation (Hall and Smith 1998) information gain-based evaluation (Forman 2003), symmetrical uncertainty-based evaluation (Hall and Smith 1999; Press et al. 1988) and wrapper-based feature subset selection (Kohavi and John 1997).

We only considered the five most predictive attributes selected/ranked by each technique. We then identified the best five predictive attributes using a round robin technique. We first created the union set of all first ranked attributes in all of the top-five lists returned by the various feature selection methods. We then found the union set of this set and all the second-ranked attributes in all of the top-five lists. We continued this procedure until the union set consisted of 5 distinct performance attributes. To understand the relative importance of the selected set of attributes, we finally ranked the top 5 selected attributes using an attribute selection technique with ranking capability (i.e., the information gain-based technique, Foreman 2003).

### Modelling and predicting performance

To model the behaviour of unsuccessful and successful riders and to understand how accurately the model could predict the riders' performances, once again, we

carried out a machine learning-based classification experiment. We made use of the same Naïve Bayes classifier used previously, using only the five most predictive attributes of performance and the unsuccessful/successful classification.

## Results

*Identification of stages within the ER*

The cluster analysis of lap velocity metrics revealed no further significant improvement in accuracy (WCSSE) is achieved by modelling the ER with any more than 2 clusters (stages). Therefore, from the perspective of physiological demands determined by the velocity of the peloton, the ER can be considered as having two stages, each with a different combination of average and variance in peloton velocity. Table 2 summarises the results of this analysis which indicates that the average point of transition between stages occurs 20 laps before the end of the ER.

**Table 2.** Characteristics of each stage of the Elimination Race. Lap velocity metrics were analyzed using unsupervised k-means clustering to determine if there was any evidence of stages. Two stages were identified using the elbow method. * Velocity in stage 2 was significantly different to stage 1.

| Lap Velocity Metric | Stage 1 | Stage 2 |
|---|---|---|
| Lap range | 1-29 | 30-50 |
| Average lap velocity (km/h) | 52.2 | 49.9* |
| SD of lap velocity (km/h) | 1.5 | 2.4 |
| Min lap velocity (km/h) | 46.8 | 33.7 |
| Max lap velocity (km/h) | 57.1 | 61.6 |

*Changes in peloton velocity throughout the Elimination Race*

Figure 1. illustrates the relatively consistent changes in peloton velocity across the four ERs we analysed. Average velocity typically remains relatively high (52.2 km/h) early in the race and progressively declines (49.9 km/h) until late in the race (prior to ~20 laps to finish). During this early stage of the race, the range of velocities within each two-lap elimination cycle, is relatively small (2.9 km/h). In the later stage of the ER (after ~20 laps to finish), peloton velocity continues to decline slightly, but there is a pronounced increase in the range of velocities (9.5 km/h).
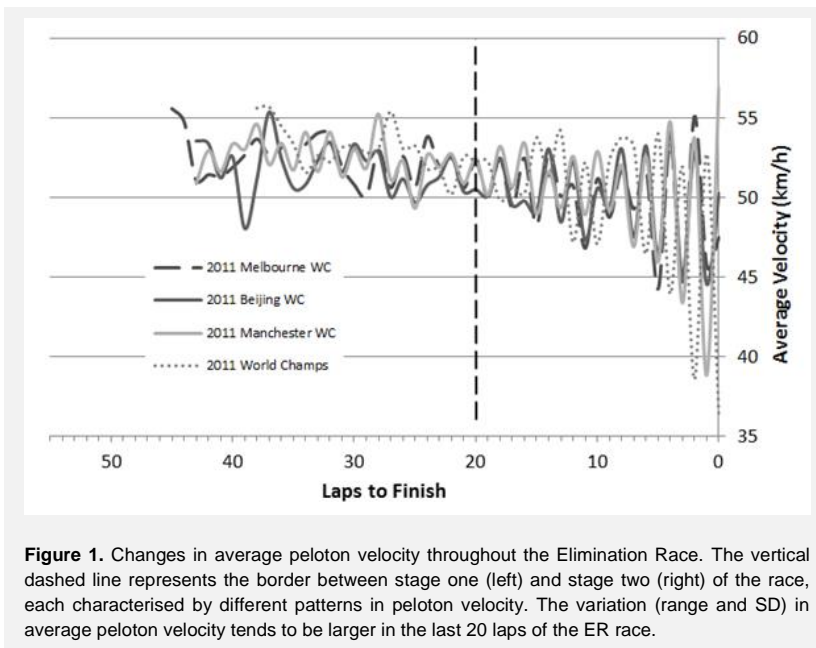


**Figure 1.** Changes in average peloton velocity throughout the Elimination Race. The vertical dashed line represents the border between stage one (left) and stage two (right) of the race, each characterised by different patterns in peloton velocity. The variation (range and SD) in average peloton velocity tends to be larger in the last 20 laps of the ER race.

*Performance modelling and prediction*

Twenty four elimination race performance attributes were analysed using a supervised Naïve Bayes classifier method. The resulting model was able to classify cyclists as being successful (finishing 6th or better) and unsuccessful (finishing worse than 6th) with an accuracy of 95.83%, which implies that the method was very reliable for performance modelling, training, or prediction purposes.

The top-five performance attributes were selected using five different feature selection methods and the results were similar, but not the same. Therefore the performance attributes with the highest aggregate ranking across the five methods were selected as the final set of top-five performance attributes; Lap 3 Distance to first, Lap 3 pos_n, Lap 3 time (s), Lap 4 x, Lap 4 pos_n.

*Performance attributes of successful and unsuccessful cyclists*

Table 3 summarises the findings of our analysis of the data we collected on the ER and it reveals the most predictive attributes of both successful and unsuccessful ER cyclists. At the point of elimination (i.e. labelled as Lap 4 of a 4 lap cycle), successful cyclists tend to be 2.47 bike lengths ahead of the back of the peloton, or when expressed relative to the changing number of cyclists in the peloton, they are mid-field (53%). In the lap before a cyclist is eliminated, successful cyclists tend to be 3 (3.09) bike lengths from the front of the peloton, whilst

**Table 3.** The final five most predictive performance attributes of successful and unsuccessful cyclists in the Elimination Race.

| Importance | Attribute | Successful cyclists (ER rank ≤ 6) | Unsuccessful cyclists (ER rank > 6) |
|---|---|---|---|
| 1st | Lap 4 x (bike lengths from the last cyclist in the peloton) | 2.47 | 0 |
| 2nd | Lap 4 pos_n (% distance from front of the peloton) | 0.53 | 0.99 |
| 3rd | Lap 3 distance to first cyclist (bike lengths) | 3.09 | 6.15 |
| 4th | Lap 3 pos_n (% distance from front of the peloton) | 0.55 | 0.81 |
| 5th | Lap 3 time (s) | 18.32 | 17.77 |

unsuccessful cyclists remain twice as far behind (6.15). When distance to the front of the peloton is normalised to the changing size of the peloton, successful cyclists tend to remain in the middle ranks (55%) and unsuccessful cyclists are at the rear of the peloton (81%). The fifth most predictive performance attribute is the lap time on the lap before elimination (Lap 3 time (s)). The results indicate a short lap time for unsuccessful cyclists and therefore a faster average velocity on this lap, in comparison to successful cyclists.

Information about the position and change of position, of the cyclists throughout the elimination cycle was also analysed using simple descriptive statistics. Figure 2 represents the positions of the first placed cyclists on elimination laps. ER race winners always remained ahead of the last row of cyclists (i.e. +1 bike lengths ahead of the last cyclist), typically between rows 1-4, and 1-3 bike widths above the inside line of the velodrome (i.e. never on the inside lane of the velodrome).

In contrast to the winning cyclists, cyclists who were one lap away from elimination, frequently appear on the last row of cyclists (i.e. 0 bike lengths from the last cyclist) and only 1-2 bike widths from the inside line of the velodrome (Figure 3).

While the horizontal (x) position in which a cyclist is eliminated is necessarily on the last row of cyclists in the peloton, it is instructive to know where on the last row they tend to be. Cyclists are most commonly eliminated closest to the inside line of the velodrome and with decreasing frequency "higher" up the velodrome (increasing bike widths above the inside line of the velodrome).



**Figure 2.** Typical positions of the first placed cyclists on an elimination lap. The tallest bars on this chart represent the positions in the peloton most frequently inhabited by first placed cyclists (i.e. in the middle of the peloton).



**Figure 3.** Typical positions of eliminated cyclists one lap before elimination. These cyclists frequently resided in the inside lane (1 bike width above the inside of the track) and on the last row of the peloton, even one lap before elimination.

## Discussion

This is the first study to present a performance analysis of the Elimination Race, which is part of the track cycling Omnium. We used a variety of machine learning approaches to analyse a large database of performance characteristics. Our results provide information about the pattern of changes in average peloton velocity throughout the ER, the performance attributes that are most predictive of success, and strategic information that can be applied by cyclists and their coaches.
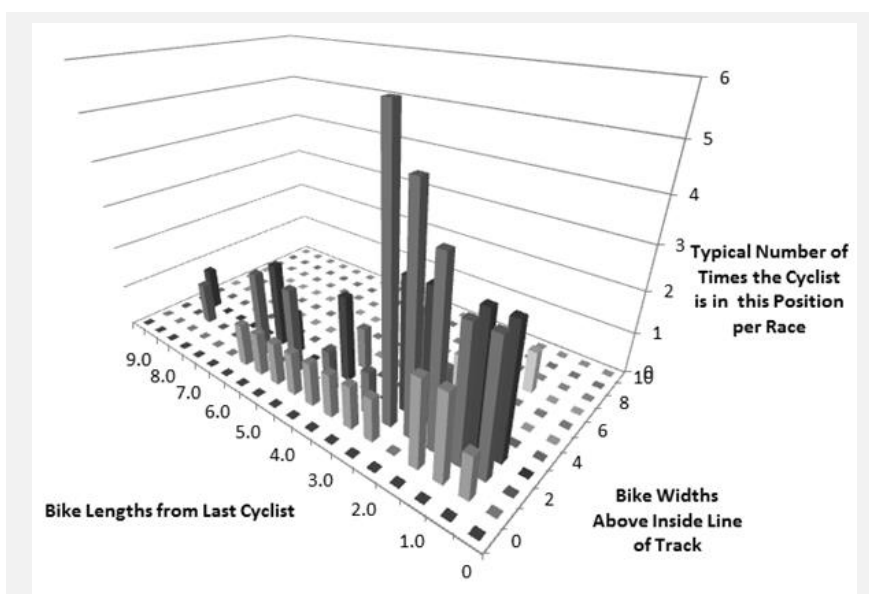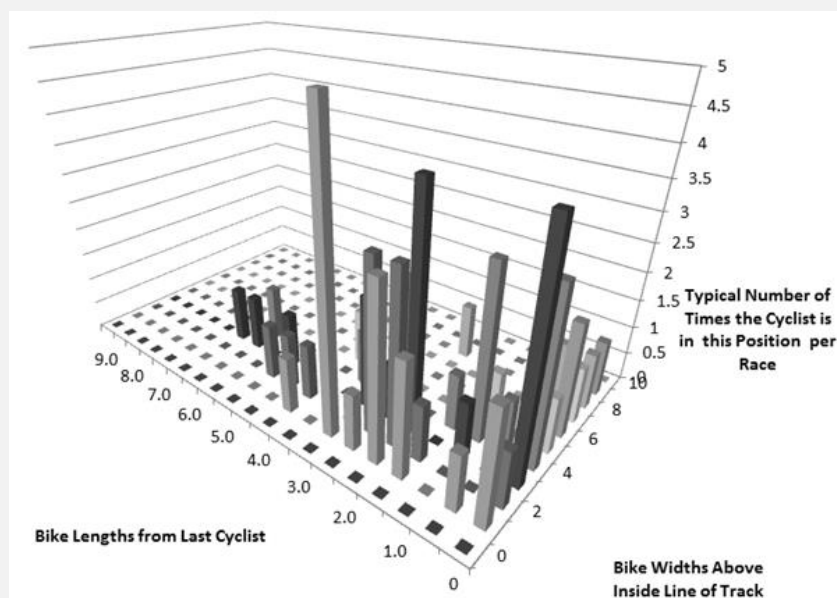
When considering the velocity demands of the ER, our analysis reveals that the ER can be considered to have two stages that intersect, on average, 20 laps before the end of the race. The first stage is characterised as having a relatively high average velocity and low variation in velocity. These features are most likely due to the large number of cyclists in the peloton being able to maintain a high velocity and a low perception of the risk of elimination leading to few attacks or changes in velocity. Average velocity also appears to progressively decline in this stage, which may indicate a combination of the decreasing number of cyclists in

the peloton and the effects of fatigue. The second stage of the ER tended to have a higher variation in velocity than the first stage. There are fewer than 10 cyclists in the peloton, in the second stage, who have all completed up to 30 laps and now face the immediate risk of elimination. So while the peloton maintain a relatively high average velocity decreases, there is an increase in the variation in velocity, as individual cyclists launch strategic attacks in order to avoid elimination and to eliminate other cyclists.

Four of the five most predictive performance attributes of the cyclists in the ER, related to their position and their change in position in the peloton. Given that we tracked most cyclists' position over four successive laps, we were able to confirm that position in the lap before elimination (lap 3) and the lap of elimination (lap 4) are the most critical factors. We also felt it was as import to identify the attributes of successful cyclists as it was unsuccessful cyclists, in order to provide advice on what to do, and what not to do.

The most powerful predictor of success in the ER was the horizontal position (x) of the cyclists on any elimination lap. Not surprisingly, unsuccessful cyclists were typically last in the peloton (i.e. on the last row of cyclists). The more revealing statistic is that the successful cyclists were not just one bike length ahead of last, nor were they at the front of the peloton. Typically, successful cyclists are most frequently found in the middle of the peloton (an average of 2.47 bike lengths from the last cyclist) and they usually avoid being on the last row of the peloton on any lap. This result also indicates that the tactic of trying to ride on the front of the peloton, which may appear to be the safest place, is not the typical choice of successful cyclists. Even on non-elimination laps, successful cyclists remain closer to the front of the peloton (3.09 bike lengths), without actually being on the front, than unsuccessful cyclists (6.15 bike lengths). Being in the middle of the peloton has the advantage of not having to work as hard as those on the front or those who make large changes to their position within each two lap elimination cycle (e.g. from the back to the front of the peloton). Successful cyclists also tend to ride in the inside three lanes (i.e. bicycle widths above the inside line of the velodrome) of the velodrome. This allows them to complete a shorter lap distance and therefore a lap lower velocity, than for cyclists who ride in the "higher" lanes.

Unsuccessful cyclists, eliminated in 7th place or worse, tended to reside in the last 20% of the peloton on the lap before they were eliminated. They also tended to ride in a higher lane than the successful cyclists (4.07 vs. 3.25 bike widths from the inside lane of the velodrome). On the lap that unsuccessful cyclists are eliminated, their position is always on the last row in the peloton, but what our analysis (Figure 3) also reveals is that many cyclists, successful and unsuccessful are eliminated closer to the inside of the track than the outside. This confirms the subjective observations of the authors that a common mistake made by cyclists is to get "caught" at the rear of the peloton on the inside lane of the velodrome. In this particular position, there are almost no opportunities to change position and avoid elimination.

The least powerful of the top five performance attributes was lap time (lap 3 time) on the lap before an elimination lap, which is also the lap that occurs immediately after an elimination lap. Unsuccessful cyclists have a lower average lap time than successful cyclists, which suggests that they are riding this lap with a higher velocity. This may be because they carry a higher velocity over the elimination line, into the following lap, than the successful cyclists. Alternatively it may be because they have to ride faster to improve their position from the rear of the group toward the front, in an attempt to avoid elimination on the following lap.

Machine learning techniques have been used to analyse the track cycling Omnium (Ofoghi et al. 2010) and other sports (Ofoghi et al. 2011). The present work illustrates the usefulness of these approaches for the provision of information that can be applied by elite athletes and coaches to describe the demands of the event/s. However, we concede that there are limitations to the present work that relate to the complex interactions between the cyclist's physiological capacity, their tactics and the subsequent demands of the race, and that our conclusions may not apply to female cyclists. Nevertheless, machine learning techniques can reveal the performance characteristics of successful athletes, both in terms of their physical aptitudes and their race tactics. Finally, there may also be scope for machine learning to be used to create a mathematical model of sports performance that can be used during an event to assist with strategic decisions.

## Practical applications

The present work provides practical information for coaches and cyclists who compete in the Elimination Race. The velocity demands of the ER vary between two stages in the race. From the start until ~20 laps to the finish, average peloton velocity is relatively high with only small variations in velocity, however in the final 20 laps, the lap-to-lap variation in average velocity is higher and therefore more demanding on cyclists. The most important performance attributes of ER cyclists relate to their position in the peloton. Successful cyclists tend to remain in the middle of the peloton and no further than 3 bike widths from the inside line of the velodrome. Unsuccessful cyclists typically reside in the last 20% of the peloton and in the higher lanes on the velodrome.

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