

Modeling the power-duration relationship the physiological and practical significance of the time constant parameter

Marton Horvath ^{1,*}, Erik P. Andersson ¹ and Dan Kuylensstierna ²

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¹ Department of Health Sciences, Mid Sweden University, Östersund, Sweden

² Department of Microelectronics and Nanoscience, Chalmers University of Technology, Gothenburg, Sweden

Correspondence

Marton Horvath

Department of Health Sciences, Mid Sweden University, Östersund, Sweden

marton.horvath@miun.se

Abstract

In this paper, we discuss the physiological and practical significance of model parameters within the framework of the three-parameter critical power model, with particular emphasis on the time constant parameter. We show that reparametrizing Morton's formula results in a parameter set in which each parameter adjusts distinct parts of the power-duration curve and carries a strong physiological interpretation. Consistent with our hypothesis, we demonstrated using a large dataset that similar curvature constant (W') values can arise from markedly different combinations of its underlying components. Therefore, we conclude that considering W' in isolation is inherently ambiguous, unless the anaerobic power capacity as well as the time constant parameter are also considered. Moreover, we argue that the three-parameter critical power model provides a framework which enables an enhanced characterization of cyclists compared with the two-parameter critical power model. Finally, we discuss the potential practical applications of our findings for coaches, practitioners as well as scientists.

Keywords

anaerobic capacity; critical power; cycling performance; three-parameter model; two-parameter model

1 Introduction

Mathematical models used for establishing the power-duration relationship have been widely investigated, with research primarily focusing on prediction accuracy and comparisons of parameter estimates between

models and numerical approaches (see e.g., Bergström et al., 2014; Drake, Finke, & Ferguson, 2024; Maturana et al., 2018). However, relatively little attention has been given to the physiological interpretation and practical significance of individual model



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parameters, particularly the time constant inherent to Morton's three-parameter critical power model.

We investigated the hypothesis that the curvature constant (W'), when derived using the three-parameter critical power model, is inherently ambiguous in isolation. Specifically, our alternative hypothesis proposed that identical W' values may emerge from various combinations of their underlying components.

2 Material and Methods

2.1 Theoretical Background

The two-parameter critical power model is a rectangular hyperbola with a positive horizontal asymptote. It is used for approximating the relationship between power output (P) and exercise duration (t) through the so-called critical power (CP) and the finite work capacity above CP (i.e., W') as

$$P(t) = CP + \frac{W'}{t}, \quad \text{Eq. (1)}$$

which formula predicts the power output corresponding to complete depletion of W' after t seconds (Monod & Scherrer, 1965).

The three-parameter critical power model extends Eq. (1) by incorporating a vertical asymptote at $t = -\tau$, thereby disallowing singularity at $t = 0$ s, and yielding a finite power intercept (i.e., P_{max}) (Morton 1996). By introducing the time constant (τ) into Eq. (1), the three-parameter critical power model can be formulated as

$$P(t) = CP + \frac{P_{an}}{1 + t/\tau}, \quad \text{Eq. (2)}$$

where P_{an} is the so-called anaerobic power capacity (i.e., $P_{max} - CP$), and τ represents the duration at which the power above CP declines to half of its initial value (Horvath et al., 2025). Thus, a larger τ results in a slower decline in the attainable power output capacity. Eq. (1) can be interpreted as an approximation of Eq.

(2) for long durations (i.e., $t \rightarrow \infty$); meanwhile, the two functions diverge away from each other at short durations (Figure 1).

Based on Eq. (2), τ can be expressed as

$$\tau = \frac{W'}{P_{an}} = \frac{W'}{P_{max} - CP}, \quad \text{Eq. (3)}$$

which indicates that the third model parameter can be interpreted as a characteristic endurance parameter relevant to a region of the intensity spectrum dominated by anaerobic energy turnover, a capability also referred to as "sprint endurance" (Aguiar et al., 2020; Jagim et al., 2013; McFarlane et al., 2001). In other words, τ quantifies how much endurance exists relative to the anaerobic power capacity, providing it with strong physiological relevance.

Normalized sensitivities of power-duration data points to each parameter in Eq. (2) can be derived using the model's partial derivatives with respect to each parameter. These sensitivities are illustrated through a realistic example in Figure 2.

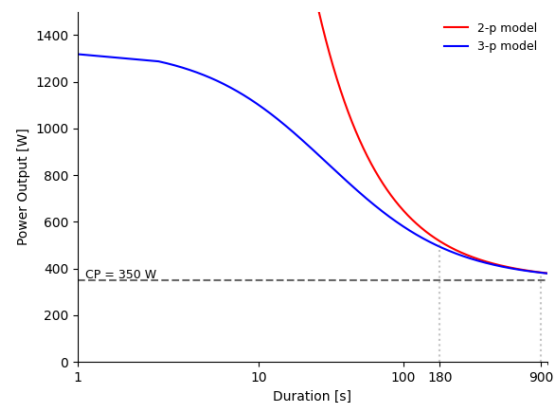


Figure 1. Illustration of the effect of introducing the time constant parameter (τ) into Eq. (1) on the power-duration curve of a theoretical cyclist with $CP = 350$ W and $W' = 30$ kJ. The two-parameter critical power model predicts a 23.8 W higher power output at a duration of 3 min compared with the three-parameter model, corresponding to a 6.8% difference relative to CP . In contrast, model selection has a negligible influence on the predicted power output at $t = 15$ min, since when $t \gg \tau$, the term $W'/(t + \tau)$ is well-approximated by W'/t .

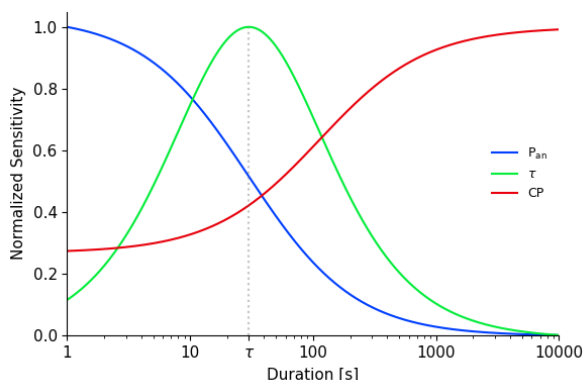


Figure 2. Illustration of the normalized parameter sensitivities of power-duration data points to model parameters CP , τ and P_{an} relative to their respective maximum values as functions of exercise duration for a theoretical cyclist ($\tau = 30$ s).

2.2 Dataset

In our analysis, we utilized publicly available data from the GoldenCheetah Open Data Project (Liversedge, 2021). This online data repository contains anonymized power output, cadence, heart rate, distance, and elevation data at both summary level and second-by-second resolution, collected over multiple years from users of the sports analysis software GoldenCheetah with their permission.

2.3 Data Processing

To enhance the robustness of our results, the used large dataset (i.e., ~100 GB of power data) was put through a cleansing process built upon the criteria previously defined by Puchowicz and Skiba (2025). First, an initial filter kept entries only if fulfilling the criteria of a minimum of one hundred ride files and a minimum duration of 2 hours for the longest file. Subsequently, any individual power-duration dataset including power recordings which exceeded mean values for the respective durations by three standard deviations was entirely excluded from the merged dataset. Maximal mean power output (MMP) profiles were extracted using the remaining ride files for each entry. Critical power model parameters were derived by fitting Eq. (2) to each obtained MMP profile via nonlinear least squares optimization implemented in Python (v3.12,

Python Software Foundation, DE) using the *SciPy* library (Virtanen et al., 2020). Based on common methodological recommendations regarding power-duration relationship establishment when using the two-parameter critical power model (Poole et al., 2016; Vanhatalo et al., 2010), the fitted duration range was restricted to the 1 s – 15 min range. Finally, to ensure that no unrealistic entry remained, the dataset was cleaned using an iterative z-score-based elimination method. Briefly, an entry was entirely excluded from the dataset if any of the parameter estimates' z-score exceeded a value of 3, which process was performed repetitively. Following this process, the final dataset included model parameter estimates for 2511 entries. All entries were checked by a researcher before further analysis.

2.4 Statistical Analysis

Linear regressions were used to quantify the proportion of variance in τ and P_{an} unexplained by W' (i.e., $1 - R^2$); furthermore, to assess the association between these two underlying parameters. Goodness of fit was assessed through calculating the coefficient of determination (R^2) and the root mean square error (*RMSE*) for each model fit. Data are presented as mean $\pm$ standard deviation (σ), unless otherwise specified.

3 Results

After accounting for the variance explained by W' in τ ($\tau = 0.6W' + 9.7$, $R^2 = 0.36$, $P < 0.001$), 64% of the variance in τ remained unexplained. Similarly, modeling P_{an} as a function of W' yielded an unexplained variance of 67% ($P_{an} = 25.0W' + 458.13$, $R^2 = 0.33$, $P < 0.001$), reflecting that P_{an} and τ are two weakly correlated parameters ($P_{an} = -11.2\tau + 1071.8$, $R^2 = 0.07$, $P < 0.001$). The ambiguity in the interpretation of W' is depicted in Figure 3, providing mechanistic proof that a given W' may result from various $P_{an} - \tau$ combinations.

Obtained CP estimates were 281.4 ± 48.9 W, meanwhile W' estimates were 16.13 ± 5.80 kJ. Histograms of CP and W' estimate distributions are shown in Figures 4 and 5, respectively. $RMSE$ and R^2 were 16.4 ± 7.0 W and 0.99 ± 0.01 , respectively.

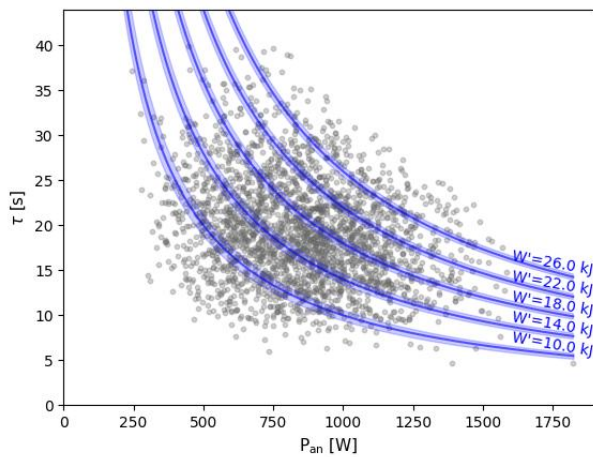


Figure 3. Illustration of W' hyperbolas $\pm 0.1\sigma_{W'}$ (blue curves and shaded areas) in the $P_{an} - \tau$ plane. Each dot represents an individual entry.

4 Discussion

Consistent with our alternative hypothesis, we demonstrated that within the framework of the three-parameter critical power model, the sole consideration of CP and W' is insufficient to characterize cyclists' performance capacities, as the interpretation of W' is inherently ambiguous unless considering τ as well as P_{an} .

Based on our results, the establishment of the power-duration relationship using the three-parameter critical power model or any other power-duration model that includes time constants (the presented analysis can be applied to e.g., the anaerobic power reserve model as well) should follow through estimating the optimal parameter set for each cyclist individually, as model parameters characterize athlete-specific features. Considering a power-duration model including only three parameters, this can typically be achieved without severe parameter correlation between the model parameters. Accordingly, procedures that impose generic time constants across

cyclists (see e.g., Leo & Mujika, 2026; Sanders et al., 2017; Sanders & Heijboer, 2019) are considered methodologically inappropriate for characterizing the individual power-duration relationship. Moreover, their use results in considerably poorer predictions than those obtained from nonlinear least squares estimation of the parameters.

Considering the physiological interpretation of our statistical investigation, it is hypothesized that riders with a given W' might demonstrate markedly different physiological characteristics (including different contributions from the anaerobic lactic and alactic energy systems), primarily determining whether they can produce high power outputs over short durations (i.e., demonstrate a relatively higher P_{an}) or can maintain a relatively higher fraction of their P_{max} during intermediate-duration maximal efforts (i.e., demonstrate a relatively higher τ).

Anecdotal evidence suggests that some sprinters are superior in “pure” sprints demanding short-duration maximal efforts on flat terrain (~10-15 s maximal intensity work), while others, despite demonstrating relatively lower maximum speeds may benefit from more demanding conditions that prolong the effort (e.g., headwinds, or low-to-moderate inclines). It is suggested that among cyclists with similar but not equal W' , those with τ above the median but with relatively lower P_{an} may be characterized as cyclists better suited for longer sprints, whereas those with P_{an} above the median but with a lower τ may be considered as “pure” sprinters. Concerning cyclists with equal W' , those with smaller τ have an advantage, as they can empty their anaerobic energy reserves relatively faster, whereas among cyclists with identical P_{an} , those with longer τ are predicted to perform better over short- and intermediate-duration efforts generally.

It is important to emphasize that τ is not a performance predictor in absolute terms. Nevertheless, when we compare cyclists of the same performance niveau with similar P_{max} and CP values (e.g., a comparison within a cohort of sprint cyclists racing at the UCI WorldTour level), it may be considered as a distinguishing parameter concerning performance characteristics and rider typology.

Although a relatively large amount of published data can be found on the power-duration relationship in elite-level cyclists (see e.g., Valenzuela et al., 2022; Mateo-March et al., 2022), considerably less information can be found on the performance characteristics of junior and youth cyclists. Studying τ typical to professional cyclists who are successful in various types of races could inform the identification of young cyclists with talent for similar types of exertion.

As part of our analysis, the three-parameter critical power model was fitted across the severe- and extreme-intensity domains, which is counterintuitive based on the distinctly different fatigue mechanisms relevant to these domains (Burnley & Jones, 2018). However, based on our results, fitting the three-parameter hyperbola can still provide reasonably good model performance until the fitted duration window is reasonably small. In contrast, fitting Eq. (2) across three intensity domains may have resulted in considerably decreased model performance.

From a physiological standpoint, applying critical power models, which represent each exercise-intensity domain separately, may provide a more appropriate representation of the underlying metabolic processes. However, such an approach could result in the comprised interpretability of model parameters and models with comprised practicality in general.

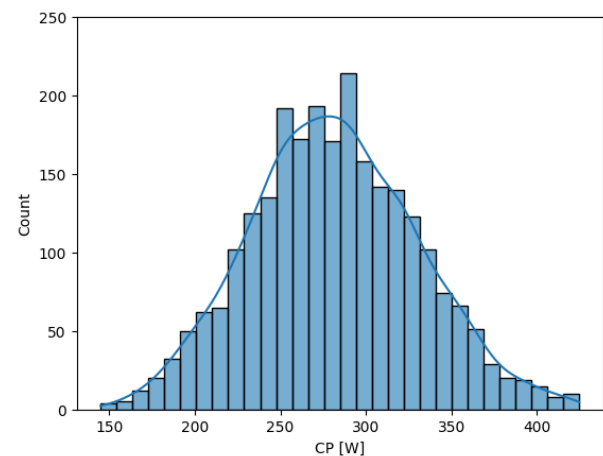


Figure 4. Histogram showing the distribution of critical power (CP) estimates.

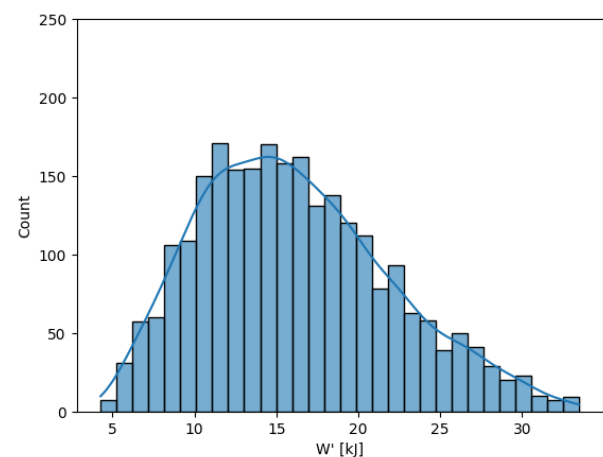


Figure 5. Histogram showing the distribution of curvature constant (W') estimates.

5 Practical Applications

Our findings underscore the importance of individually estimating the time constant parameter when characterizing the performance capacity of cyclists. Furthermore, studies utilizing power-duration models which consider time scales should report all model parameters considered, as omitting such model parameters may obscure important characteristics of the power-duration relationship. The present findings may also have practical applications for talent identification, particularly when assessing performance potential in short- and intermediate-duration efforts.

6 Conclusions

The present study provides empirical evidence that a given W' may emerge from various combinations of its underlying components (i.e., P_{an} and τ). In the framework of the three-parameter critical power model, cyclists demonstrating similar W' values are distinguished by the proportional contribution of these components, reflecting underlying physiology, and fundamentally determining performance profiles.

Data Availability: Raw datasets used in this study are publicly available as detailed in the main text. Datasets used for the main analyses will be made available by the corresponding author upon reasonable request.

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Conflicts of Interest: The authors declare no conflict of interest.

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